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Advanced Statistical Physics - Problem set 13

Summer Terms 2022

Hand in: Hand in tasks marked with * to mailbox no. (43) inside ITP room 105b by Friday 8.07. at 9:15 am.

21. The renormalization group of the Ising model* *2+2+3+2+2+2+2 Points*

The differential recursion relations for temperature T, and magnetic field h, of the Ising model in $d = 1 + \epsilon$ dimensions are (for $b = e^{\ell}$)

$$\begin{cases} \frac{dT}{d\ell} & = -\epsilon\, T + T^2/2 \\ \frac{dh}{d\ell} & = dh \end{cases} .$$

(a) Sketch the renormalization group flows in the (T, h)-plane (for $\epsilon > 0$), marking the fixed points along the $h = 0$ axis.

(b) Calculate the eigenvalues y_t and y_h, at the critical fixed point, to order of ϵ.

(c) Starting from the relation governing the change of the correlation length ξ under renormalization, show that

$$\xi(h, t) = t^{-\nu}\, g_{\xi}\left(h/|t|^{\Delta}\right) \ , \ \ t = \frac{T}{T_c} - 1 \ ,$$

find the exponents Δ and ν

(d) Use a hyperscaling relation to find the singular part of the free energy $f_{sing}(t, h)$, and hence the heat capacity exponent α.

(e) Find the exponents β and γ for the singular behaviors of the magnetization and susceptibility, respectively.

(f) Starting with the relation between susceptibility and correlations of local magnetizations, calculate the exponent η for the critical correlations $\left(\langle m(0)m(x)\rangle \sim |x|^{-(d-2+\eta)}\right)$.

(g) How does the correlation length diverge as $T \to 0$ (along $h = 0$) for $d = 1$?